OFF-SHELL NN POTENTIAL AND TRITON BINDING ENERGY [1]

Y. Song and R. Machleidt
Department of Physics, University of Idaho, Moscow, ID 83843, U.S.A.

SUMMARY AND CONCLUSIONS

The (*nonlocal*) Bonn-B potential predicts 8.0 MeV binding energy for the triton (in a charge-dependent 34-channel Faddeev calculation) which is *about 0.4 MeV more than* the predictions by *local* NN potentials. We pin down origin and size of the nonlocality in the Bonn potential, in analytic and numeric form. The nonlocality is due to the use of the correct off-shell Feynman amplitude of one-boson-exchange avoiding the commonly used on-shell approximations which yield the local potentials. We also illustrate how this off-shell behavior leads to more binding energy. We emphasize that the increased binding energy is not due to on-shell differences (differences in the fit of the NN data or phase shifts). In particular, the Bonn-B potential reproduces accurately the ϵ_1 mixing parameter up to 350 MeV as determined in the recent Nijmegen multi-energy NN phase-shift analysis. Adding the relativistic effect from the relativistic nucleon propagators in the Faddeev equations, brings the Bonn-B result up to 8.2 MeV triton binding [1]. This leaves a difference of only 0.3 MeV to experiment, which may possibly be explained by refinements in the treatment of relativity and the inclusion of other nonlocalities (e. g., quark-gluon exchange at short range). Thus, it is conceivable that a realistic NN potential which describes the NN data up to 300 MeV correctly may explain the triton binding energy without recourse to 3-N forces; relativity would play a major role for this result.

INTRODUCTION

Recently it has been shown that *local NN potentials* lead to a triton binding energy of 7.62 ± 0.01 MeV [2], i. e., they *underbind the triton by 0.86 MeV*. On the other hand, it is well known that the nuclear force must contain nonlocalities, since any more fundamental mechanism for creating the

[1]Contributed Paper submitted to the *14th International Conference on Few Body Problems in Physics*, Williamsburg, Virginia, USA, May 26-31, 1994.

nuclear force generates a nonlocal interaction. Such nonlocalities may increase binding energy predictions for systems of three or more nucleons. An example is the Bonn-B potential [3] which predicts 7.97 MeV binding energy for the triton in a charge-dependent 34-channel Faddeev calculation [4].

Before one can pin down off-shell effects, it is absolutely crucial to make sure that the potential under consideration reproduces accurately the on-shell NN data. Of particular importance is here the ϵ_1 mixing parameter which is a measure for the on-shell tensor force strength. Figure 1 shows that the the Bonn-B potentials predicts ϵ_1 in accurate agreement with the most recent Nijmegen multi-energy NN phase-shift analysis [6].

OFF-SHELL POTENTIAL AND NN t-MATRIX

Predictions for NN observables may be based upon the on-shell two-nucleon t-matrix derived from a given NN potential V. The off-shell NN t-matrix is input for momentum-space Faddeev calculations of the three-nucleon system. The calculation of the t-matrix always involves the NN potential on- and off-(the-energy-)shell. We illustrate this for the example of the partial-wave t-matrix, $t^{JST}_{L'L}$, in the $^3S_1$ two-nucleon state, which is given by

$$\begin{aligned} t^{110}_{00}(q',q;E) &= V^{110}_{00}(q',q) - \int_0^\infty k^2 dk V^{110}_{00}(q',k) \frac{M}{k^2 - ME - i\epsilon} t^{110}_{00}(k,q;E) \\ &\quad - \int_0^\infty k^2 dk V^{110}_{02}(q',k) \frac{M}{k^2 - ME - i\epsilon} t^{110}_{20}(k,q;E) \; . \qquad (1) \end{aligned}$$

For free-space NN scattering, $E = q_0^2/M$ with M the nucleon mass and q_0 the c.m. on-shell momentum which is related to the lab. energy by $E_{lab} = 2q_0^2/M$. Notice that in the integral terms the potential contributes essentially off-shell. In general, the second integral which involves V^{110}_{02} (tensor force) is much larger than the first integral that involves V^{110}_{00} (central force). Therefore, we will focus here on the second integral term and the tensor force.

In Fig. 2, we show the $^3S_1$-$^3D_1$ potential matrix element, $-V^{110}_{02}(q_0,k)$, for Paris [5] and Bonn [3]. The momentum q_0 is held fixed at 153 MeV which corresponds to a lab. energy of 50 MeV. The abscissa, k, is the variable over which the integrations are performed in the above equation. It is seen that, particularly for large off-shell momenta, the Bonn-B potential is substantially smaller than the Paris potential: the Bonn-B potential has clearly a weaker

off-shell tensor force than the Paris potential. As a consequence of this, the Paris potential produces a much larger integral term in Eq. (1) than the Bonn potential; and it is this integral term that is subject to quenching in few- and many-body calculations. In the three-body Faddeev equations, the t-matrix is fully off-shell and E is negative; the negative E reduces the magnitude of the integral term. The larger the term, the larger the quenching. Since this integral term is attractive, the quenching is a repulsive effect. Thus, large off-shell potentials, implying a large integral term, yield less attraction in three- and many-body problems. This explains why the Paris potential predicts less triton binding energy than the Bonn potential.

THE ORIGIN OF OFF-SHELL DIFFERENCES

The Bonn potential is defined in terms of the full, relativistic Feynman amplitudes of one-boson-exchange, which are non-local expressions. On the other hand, the Paris potential is (apart from a $\mathbf{p}^2$-term in the central force) a local potential. The local approximation of the pion tensor potential, which is used in the Paris potential as well as in any other local r-space potential, has the familiar form,

$$ {}^{\pi}\tilde{V}_T(r) = -\frac{g_\pi^2}{4\pi}\left(\frac{m_\pi}{2M}\right)^2 \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r} S_{12} \; . \qquad (2) $$

The transformation of this local potential into momentum space yields for the ${}^3S_1$-${}^3D_1$ amplitude

$$ {}^{\pi}\tilde{V}_{02}^{110}(q_0,k) = -\frac{g_\pi^2}{4\pi}\frac{\sqrt{8}}{4\pi M^2 q_0 k}[q_0^2 Q_2(z) - 2q_0 k Q_1(z) + k^2 Q_0(z)] \qquad (3) $$

with Q_L Legendre funct. of the 2. kind and $z \equiv (q_0^2 + k^2 + m_\pi^2)/(2q_0 k)$. The original ${}^3S_1$-${}^3D_1$ transition potential as it results from the relativistic one-pion-exchange Feynman amplitude is

$$ \begin{aligned} {}^{\pi}V_{02}^{110}(q_0,k) \;=\; & -\frac{g_\pi^2}{4\pi}\frac{\sqrt{8}}{4\pi M^2 q_0 k}[(E_{q_0} - M)(E_k + M)Q_2(z) - 2q_0 k Q_1(z) \\ & + (E_{q_0} + M)(E_k - M)Q_0(z)] \end{aligned} \qquad (4) $$

with $E_{q_0} \equiv \sqrt{M^2 + q_0^2}$ and $E_k \equiv \sqrt{M^2 + k^2}$. Expanding these roots in terms of q_0^2/M and k^2/M yields

$$
\begin{aligned}
{}^{\pi}V_{02}^{110}(q_0, k) \;\; \approx \;\; & -\frac{g_\pi^2}{4\pi} \frac{\sqrt{8}}{4\pi M^2 q_0 k} \left[\left(q_0^2 + \frac{q_0^2 k^2}{4M^2} - \frac{q_0^4}{4M^2} \cdots \right) Q_2(z) - 2 q_0 k Q_1(z) \right. \\
& \left. + \left(k^2 + \frac{q_0^2 k^2}{4M^2} - \frac{k^4}{4M^2} \cdots \right) Q_0(z) \right] \; . \qquad (5)
\end{aligned}
$$

Keeping terms up to momentum squared, leads to the local approximation Eq. (3). The largest term in the next order is $-k^2/(4M^2)Q_0(z)$ which damps the tensor potential off-shell.

The thin short-dashed curve in Fig. 2 shows what is obtained when in the Bonn-B potential the pion tensor force is made local and the dotted curve results when both π and ρ are local. The latter curve is very close the Paris curve (thick long-dashed line), which explains the origin for the differences between the Paris and Bonn potentials. Another source for nonlocality in the Bonn potential is the factor $M/\sqrt{E_{q_0} E_k}$ which is applied to the Feynman amplitude to define the potential. The nonlocality created by this factor has been investigated in great detail by Glöckle and Witala [7]. This factor is included in the solid, short-dashed, and dotted curves in Fig. 2. When this factor is omitted in the latter case, the widely-spaced dots are obtained, which demonstrates an additional source of nonlocality in the Bonn potential.

This work was supported in part by NSF-Grant PHY-9211607.

REFERENCES

1. F. Sammarruca, D. P. Xu, and R. Machleidt, Phys. Rev. C **46**, 1636 (1992).
2. J. L. Friar *et al.*, Phys. Lett. **B311**, 4 (1993).
3. R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
4. D. Y. Song and R. Machleidt, unpublished.
5. M. Lacombe *et al.*, Phys. Rev. C **21**, 861 (1980).
6. V. G. J. Stoks *et al.*, Phys. Rev. C **48**, 792 (1993).
7. W. Glöckle and H. Kamada, *Nonlocalities in NN forces defined through the Blankenbecler-Sugar equation*, preprint, University of Bochum (1994).

Figure Captions

Figure 1. The ϵ_1 mixing parameter below 350 MeV as predicted by the Bonn-B [3] (solid line) and the Paris [5] (dashed) potentials. The solid dots represent the Nijmegen multi-energy NN phase shift analysis [6].

Figure 2. Half off-shell $^3S_1$-$^3D_1$ transition potential of the Bonn-B [3] (solid line) and the Paris [5] (thick dashed line) potentials. The thin short-dashed and dotted lines are obtained when in the Bonn-B potential the local approximation is used for the π and for both π and ρ, respectively. When in the latter case also the $M/\sqrt{E_{q_0}E_k}$ factor is left out, the widely-spaced dots are obtained. The solid dot is the on-shell point ($k = q_0$) at which all curves are the same.

The figures are available upon request from

MACHLEID@TAMALUIT.PHYS.UIDAHO.EDU

Please include your FAX number or mailing address with your request.